

Flexible Deviations from FIRE in the Sequence Space

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Outline

Introduction and Research Aims

Flexible Deviations from FIRE: Theoretical Framework

Kohlhas and Warther Reduced Form Expectations

The \mathcal{K} mapping in the Sequence Space

DSGE Sequence Space Solvent: the Behavioural Jacobian

Sequence-Space Implementation and Applications in HANK

One-Asset HANK

Two-Asset HANK

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Introduction

Background and Research Questions

- ▶ Since Lucas, *Full Information and Rational Expectations* have been a cornerstone of DSGE. However, empirical evidence decisively rejects the hypothesis of (FIRE) among agents.
- ▶ **Q1.** How can we depart from FIRE in a way that is (a) micro-founded and (b) consistent with such empirical evidence, as well as (c) flexible enough to handle a number of expectation formation models?
- ▶ For HANK to become the workhorse framework of macroeconomics and policy, it must match key aspects of macro transmission: *micro-jumps* (iMPCs) and *macro-humps* (Auclert et al. 2021).
- ▶ **Q2.** Can our model of flexible deviations from FIRE model reduce persistence and variance of shocks needed to match empirical transmission relative to extant frameworks (i.e. Sticky expectations, cf. [Albuquerque et al., 2025](#))? ▶ MJMH

Main Contributions

- ▶ Propose a framework for departing from FIRE building on the reduced-form model in [Kohlhas and Warther \(2021\)](#). Several benefits:
 1. Integrable with micro and professional forecasters survey data evidence: overreaction to current conditions and underreaction to (public) news.
 2. Micro-founded in asymmetric attention mechanisms.
 3. Flexible, nesting cases including FIRE, *Sticky* expectations ([Auclert et al., 2021](#); [Gabaix, 2020](#); [Coibion and Gorodnichenko, 2015](#)), and *Diagnostic* expectations ([Bordalo et al. 2018](#)).
 4. Under suitable parametrisations, near-rationality.
 5. Can apply to other agents in the economy, not just households.
- ▶ We show how to implement this *Reduced Form* (RF) expectation process in the Sequence Space in a way equivalent to the computational framework for deviations from FIRE proposed by [Bardoczy and Guerreiro \(2024\)](#).

Cont'd

- ▶ We develop an application to HANK models in the sequence space, covering implications for **macro-transmission and dynamics**:
 1. Develop and solve under alternative expectation processes (FIRE, Sticky, RF) a One-Asset and a Two-Asset HANK, replicating closely features of [Auclert et al. 2021](#).
 2. Micro-jumps and Macro-humps under alternative expectation processes, and how RF mitigates the dampening effect of sticky expectations on amplification.
- ▶ ... and **business cycle estimation** questions:
 1. How strong a departure from FIRE and in what direction: *partial information* estimation of the RF expectations parameters by IRF-matching to the empirical IRFs by [Romer and Romer \(2024\)](#) and [Bauer and Swanson \(2023\)](#)
 2. Estimation and amplification questions: *Likelihood-Based* estimation of the aggregate shock processes as in [Smets and Wouters \(2007\)](#) across FIRE, Sticky, and RF expectations.

Outline

1. Theoretical framework and the Kohlhas-Warther (KW) mapping $\mathcal{K} : X \rightarrow X^e$
2. Implementation in the Sequence Space and constructing the Sequence-Space Jacobians.
3. Model(s) description: One-Asset and Two-Asset HANK
4. Partial and Full-Info estimation strategies
5. Main results
6. Coming next...

Flexible Deviations from FIRE: Theoretical Framework

Kohlhas and Warther Reduced Form Expectations

Consider the model for the i -th individual forecast error on an aggregate input x_{t+k} , k periods ahead time t when the forecast $f_{i,t}x_{t+k}$ is formed.

$$x_{t+k} - f_{it}x_{t+k} = \alpha_i + \underbrace{\gamma x_t}_{\text{Extrapolation}} + \underbrace{\delta(\bar{f}_t x_{t+k} - \bar{f}_{t-1} x_{t+k})}_{\text{News Revision}} + \varepsilon_{i,t|t+k}$$

- ▶ Where $\bar{f}_t x_{t+k} = \int_j f_{j,t} x_{t+k} P(dj)$ the average forecast.
- ▶ Coefficients γ and δ denote *extrapolation* and (*public*) *news revision*. Empirical estimates with US SPF data $\gamma < 0, \delta > 0 \rightarrow$ consistent with IRF matching from HANK model?
- ▶ Note model makes sense ex-post, i.e. conditional on the info set \mathcal{F}_{t+k} (and super-sets)... sequence space is a natural environment.
- ▶ Time-invariant here... but not needed as we show.


Cont'd

- ▶ Nests multiple models, including alternative to the one we propose, remaining moot/agnostic on sources of informational friction
 1. Purely sticky expectations as in [Coibion & Gorodnichenko 2015](#); [Auclert et al., 2020](#): $\delta > 0$, $\gamma = 0$.
 2. Purely extrapolative or *diagnostic* expectations in the sense of [Bordalo et al. \(2018\)](#): $\gamma \neq 0$, $\delta = 0$
 3. In *RF*, coefficient signs micro-founded in limited attention mechanisms and noisy signals in unobserved component SSMs.
- ▶ Key next steps for implementation in DSGE models in the Sequence-Space:
 1. Getting the mapping \mathcal{K} from actual realised inputs $X \in \mathcal{X}^T$ to agents' forecasts $X^{e,\tau} \in \mathcal{X}^{T \times T}$.
 2. Using this to pin down the Sequence-Space Jacobian with behavioural expectations based on the FIRE Jacobian.

The \mathcal{K} Mapping as a Recursive LoM for the Forecast

- ▶ Taking expectations with respect to the measure of agents i , and re-arranging, $\forall t, k$:

$$\bar{f}_t x_{t+k} = c + \frac{1}{1+\delta} (\delta \bar{f}_{t-1} x_{t+k} + x_{t+k} - \gamma x_t)$$

- ▶ Under a set of technical assumptions  view, it can be shown that at a 1st order perturbation expansion around the steady state:


$$\bar{f}_t x_{t+k} = x_{ss} + \frac{1}{1+\delta} (\delta \bar{f}_{t-1} dx_{t+k} - \gamma dx_t)$$

- ▶ In Sequence Space, forecast evolution governed by a *deterministic* recursive Law of Motion \rightarrow mapping $\mathcal{K} : X \rightarrow X^{e,t}$

Cont'd

- Made explicit by solving, for all $t, k \leq T$, the difference equation backwards up to the origin of the transition $t = 0$:

$$dx_{t+k}^{e,t} := d\bar{f}_t x_{t+k} = \underbrace{\frac{1}{1+\delta} \sum_{j=0}^t \left(\frac{\delta}{1+\delta} \right)^{t-j} dx_{t+k}}_{\text{News Effect (underreaction)}} - \underbrace{\gamma \sum_{j=0}^t \left(\frac{1}{1+\delta} \right)^{t-j} dx_j}_{\text{Extrapolation Effect (Overreaction)}}$$

- **Intuition.** In Sequence Space, all uncertainty on future paths gets resolved at time zero. Shocked input dx_{t+k} becomes part of the *full information set*. How does this transmit to forecasts under FIRE and RF Models?
 1. News Effect with $\delta > 0$: delayed response... macro humps as in Sticky Expectations
 2. Extrapolation Effect with $\gamma < 0$: amplification/larger response on impact.
- Linear mapping... fast implementation in DSGE solvents through manipulation of the FIRE Jacobian  Mapping

$$d\mathbf{X}^{e,\tau} = \mathcal{K}_\tau d\mathbf{X}$$

RF Jacobian as a function of the FIRE Jacobian

- ▶ We employ a procedure that we show to be equivalent to [Bardoczy and Guerreiro' \(2024\)](#) generalisation of the SSJ method proposed for FIRE environments by [Auclert et al. \(2021\)](#).
- ▶ DSGE Model cast as a Sequence-Space Equilibrium in DAG form. Square "Target Block":

$$\begin{aligned} \mathbf{F}(\mathbf{U}, \mathbf{Z}) &= \mathbf{0} \\ d\mathbf{U} &= \underbrace{\mathbf{F}_U^{-1} \mathbf{F}_Z}_{\text{Gen. Eq. Jacobian } G} d\mathbf{Z} \\ d\mathbf{X} &= \underbrace{\mathbf{S}_X^{-1} \mathbf{S}_U}_{\text{Part. Eq. Jacobian wrt } X} d\mathbf{X} + \underbrace{\mathbf{S}_X^{-1} \mathbf{S}_Z}_{\text{Part Eq. Jacobian wrt } Z} d\mathbf{Z} \end{aligned}$$

- ▶ DAG formulation \rightarrow break G —finding problem into sequence of smaller problems solving for Partial Eq. Jacobian.
- ▶ In HANK, key object is response of (heterogeneous) agents to shocks to the inputs: (heterogeneous) agent block Jacobian (as $dX_t \rightarrow 0$):

$$\tilde{\mathbf{J}} = [dY_t/dX_s]_{(t,s) \in T^2}$$

- ▶ Fast construction through Fake-News matrix...

Cont'd

- ▶ Let's tie things together. Deviations from FIRE \rightarrow *forecast aggregate inputs* are inputs to the household DP problem.
- ▶ To derive the Jacobian $\tilde{\mathbf{J}}$, procedure is equivalent to [Bardoczy and Guerreiro' \(2024\)](#) generalisation of the SSJ method to non-FIRE environments.
- ▶ Under linearity of the Kohlhas-Warther mapping, the Jacobian $\tilde{\mathbf{J}}$ can as a function of \mathbf{J} and parameters of the \mathcal{K} mapping:

$$\tilde{\mathbf{J}} = \mathbf{J}\mathcal{K}_0 + \sum_{h=1}^T R_h(\mathcal{K}_h - \mathcal{K}_{h-1})$$

where $R_h = \begin{pmatrix} \mathbf{0} & \mathbf{0}'_h \\ \mathbf{0}_h & \mathbf{J} \end{pmatrix}$

- ▶ What's the intuition?  Faster

Sequence-Space DSGE Solvent with RF Expectations

- ▶ Pulling together, the proposed DSGE solvent with RF expectations (and nested cases!) is:

$$d\mathbf{U} = \mathbf{F}_U^{-1} \mathbf{F}_Z d\mathbf{Z}$$

$$d\mathbf{U} = [\tilde{\mathbf{J}} \mathbf{S}_X^{-1} \mathbf{S}_U]^{-1} \mathbf{F}_Z d\mathbf{Z}$$

$$d\mathbf{U} = \underbrace{\left[\left(\mathbf{J} \mathcal{K}_0 + \sum_{h=1}^T R_h (\mathcal{K}_h - \mathcal{K}_{h-1}) \right) \mathbf{S}_X^{-1} \mathbf{S}_U \right]^{-1}}_{\text{Gen. Eq. Jacobian with Flexible Deviations from FIRE}} \mathbf{F}_Z d\mathbf{Z}$$

- ▶ Solvent collapses to FIRE solvent when consistently we set

$$\mathcal{K}_0 = \mathcal{K}_s = \mathbf{I}, \forall s.$$

- ▶ **Next.** Application to transmission and estimation in HANK environments

Applications to HANK in the Sequence Space

Overview

We propose a number of applications of the expectations model and solvent to transmission and estimation in HANK environments. In particular:

1. Role of behavioural parameters in relative reaction to and amplification of aggregate shocks:
 - ▶ matching *micro-jumps* (iMPCs) and *macro-humps*.
 - ▶ Consistency with empirical estimates $\gamma < 0, \delta > 0$
 - ▶ Decomposition of Business Cycle into extrapolative and news-revision components.
2. Performance in matching empirical IRFs relative to nested Sticky and FIRE models
 - ▶ Romer & Romer (2004) Jordà LPs
 - ▶ Bauer and Swanson (2023) Proxy VAR
3. Business Cycle estimation: explaining variance of macro-series with [Smets and Wouters \(2007\)](#) shocks specification.

HANK Environments

- ▶ Applications currently explored in a set of HANK models:
 1. One-Asset HANK model with aggregate investment and a consolidated asset in the spirit of MHMJ [Auclert et al. \(2020\)](#) and [Auclert et al. \(2024\)](#)
 2. A "Two-Asset" HANK model with permanent heterogeneity, aggregate investment, and illiquid assets closely replicating MHMJ [Auclert et al. \(2021\)](#)
 3. The UK HANK model with housing by [Albuquerque et al. \(2025\)](#)
- ▶ Model outline and estimation methods
- ▶ Preliminary Results and Findings

The MJMH *Two-Asset* HANK

Households

- ▶ Permanent heterogeneity on HH side to match features of wealth and income inequality in US → key for micro-jumps...
 1. Six groups or fixed types with exogenous density (btm 50% to top 5% matching quantiles of the illiquid distribution of wealth (US SFC 2013).
 2. Group-Level differences in steady state (1) illiquid asset shares, (2) discount factors, (3) average skill level/efficient labour units.

For each group $j \in \{1, \dots, 6\}$:

$$V_t(l_{t-1}, a_{t-1}, z_t, j_t; \mathbf{X}_t) = u(c_t, n_t) + \beta_j \mathbb{E}_t V_{t+1}(l_t, a_t, z_{t+1}, j_{t+1}; \mathbf{X}_{t+1})$$

$$c_t + l_t = (1 + r_t^l) a_{t-1} + (1 - \tau_t) w_t n_t \bar{z}_j \tilde{z} + x_t + d_t(a_{t-1}, r_t^a, j)$$

$$a_t = (1 + r_t) a_{t-1} - d_t(a_{t-1}, r_t, j)$$

$$d_t(a_{t-1}, r_t, j) = \frac{r_{ss}^a}{1 + r_{ss}^a} (1 + r_t^a) a_{t-1} + \chi [(1 + r_t) a_{t-1} - (1 + r_{ss}^a) \bar{a}_j]$$


- ▶ Idiosyncratic Transition Matrix/Markov Kernel?

$$\Pi((z_t, j_t), (z_{t+1}, j_{t+1}))$$

Aggregate Investment

- ▶ Convex (deadweight) adjustment costs and time-to-build.
- ▶ *Capital firm (K-Firm)* plans optimal investment at t to be carried out tomorrow, and financed out of revenue from renting out stock final good firm (Y-firm).
- ▶ Maximises shareholder value / value of the firm.
 1. $K_t = I_{t-1} + (1 - \delta)K_{t-1}$
 2. Optimal I_t and Tobin's Q pinned down by Q-theory equations.


Financial Side and Asset Markets

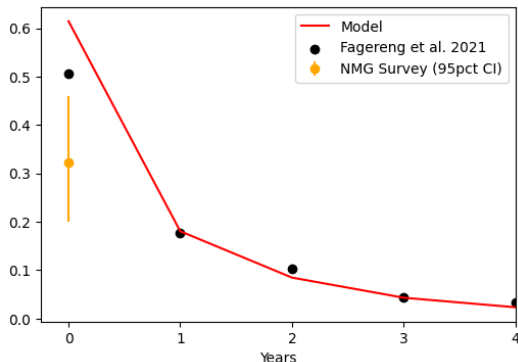
- ▶ Mutual fund collects liquid and illiquid deposits, with (deadweight) intermediation costs on former.
- ▶ Invests in gov't bonds, and shares in K -firm and Y -firm  Fund

Calibration

- ▶ We follow the (quarterly) calibration of untargeted steady state parameters and aggregates in [Auclert et al. 2020](#) – others estimated by IRF matching ([Eichengreen, Christiano, and Evans, 2005](#)).
- ▶ HH: income process uses the **Kapman-Moll-Violante** spec, suitably rescaled to match:
 1. Income shares of the illiquid asset groups ([2013 US SFC](#))
 2. Progressive income taxation [HSV, 2017](#)
- ▶ Targeted parameters include discount factor of bottom 50% (Group 1) to match illiquid asset holding shares, and K-elasticity of output to match US capital stock at asset market clearing.


Steady State

- ▶ Aggregate targets replicate the MJMH steady state almost perfectly.  Steady State
- ▶ **Micro Jumps:** iMPCs



- ▶ **Next.** Transmission and Dynamics under RF and nested models with estimated parameters (23).

Estimation Methods and Routines: Partial Info Estimation

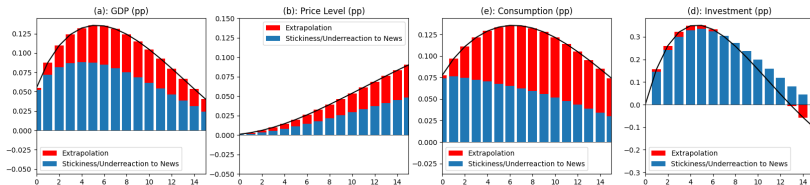
- ▶ For now, two-step procedure as in MJMH.
- ▶ Partial Information Estimation for IRF Matching
 1. Romer & Romer (2004) MP Shocks
 - 1.1 Jordà Linear Local Projections → "Empirical IRFs"
 - 1.2 Target "dynamic" parameters, including behavioural ones, to minimise loss of models IRFs relative to LP IRFs.
 - 1.3 Redform and nested Sticky, FIRE models (as restrictions)
 2. Ongoing... Bauer & Swanson (2023) MP Shocks  BS IRFs
 - 2.1 Proxy-VAR Model
 - 2.2 As Above
- ▶ **Targeted Dynamic Parameters:** Extrapolation γ , News Revision δ , Taylor Rule Inertia, Adj. Costs curvature, calvo probability NKPC-p, calvo probability NKPC-w, Taylor Rule coefficient.
- ▶ Groundwork for the BC decomposition exercise.

Estimation Methods and Routines: Business Cycle Estimation

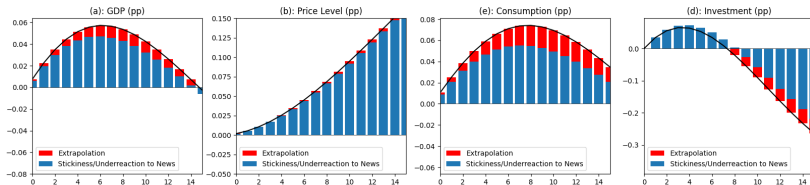
- ▶ Full Information or Likelihood-Based Estimation against Business Cycle Data for Smets & Wouters (2007) "VARMA" aggregate shocks
 1. HH Discount factor (AR1), risk premium (AR1), monetary policy shock (AR1), fiscal expenditure shock (AR1), TFP shock (AR1), NKPC-p cost-push shock (ARMA11), NKPC-w cost push shock (ARMA11)
 2. AR and MA coefficients + standard deviations = 16 parameters.
 3. Priors identical to [Smets and Wouters \(2007\)](#) and [Auclert et al. \(2021\)](#)
- ▶ Estimation framework "semi-Bayesian": maximisation of Posterior-Likelihood $P(\Theta|\mathbf{Y}) \propto \mathcal{L}(\mathbf{Y}|\Theta)P(\Theta) \rightarrow$ "Mode-Search" + Laplace Approximation for credible sets. ▶ LLF
- ▶ Numerical optimisation: *Nelder-Mead*
- ▶ Probabilistic or MCMC-style optimisation: *Simulated Annealing*

Main (Current) Results from Estimation and Applications

Business Cycle Decomposition: Macro-Humps



Expansionary MP Shock



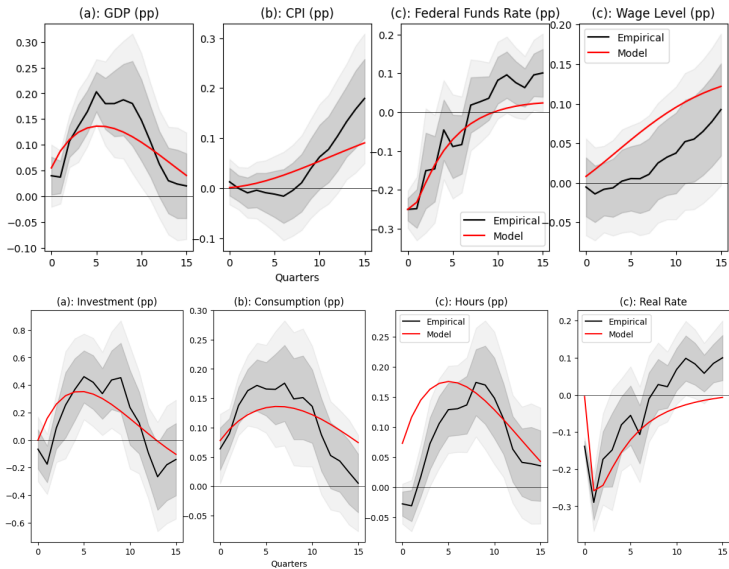
Expansionary TFP Shock

IRF Matching: Dynamic Parameter Estimates

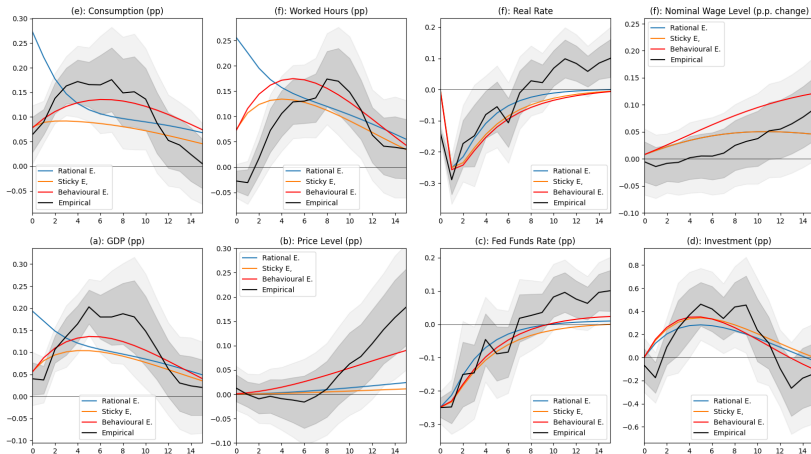
- ▶ Coefficient signs in line with micro-foundations and SPF empirical evidence in [Kohlhas & Warther \(2021\)](#)
- ▶ Interaction between Extrapolation and News channels

Dynamic Parameter	RF	Sticky	FIRE
Extrapolation γ	-0.471	-	-
News Revision δ	16.649	9.209	-
Adj. Cost Curvature	5.015	5.036	5.011
Taylor Rule Inertia	0.763	0.764	0.687
Taylor Rule Coeff.	1.057	1.645	1.305
Calvo-Prob (Prices)	0.956	0.987	0.990
Calvo-Prob (Wage)	0.950	0.941	0.916
IRF Loss	0.706	0.742	1.132

IRF Matching to Romer & Romer



Cont'd



Smets & Wouters Shocks Estimation

Shock Process	Parameter	RF	Sticky	Priors
Discount Factor	AR	0.216	0.025	Beta(0.5, 0.2)
	sd	2.443	4.300	Invgamma(0.1, 2)
Risk Premium	AR	0.588	0.596	Beta(0.5, 0.2)
	sd	6.036	4.807	Invgamma(0.1, 2)
Monetary Policy	AR	0.427	0.670	Beta(0.5, 0.2)
	sd	0.010	0.009	Invgamma(0.1, 2)
Fiscal Expenditure	AR	0.974	0.904	Beta(0.5, 0.2)
	sd	0.301	0.381	Invgamma(0.1, 2)
Productivity (TFP)	AR	0.220	0.424	Beta(0.5, 0.2)
	sd	0.001	0.168	Invgamma(0.1, 2)
Cost-Push NKPC-p	AR	0.637	0.956	Beta(0.5, 0.2)
	sd	0.001	0.0004	Invgamma(0.1, 2)
	MA	0.861	0.613	Beta(0.5, 0.2)
Cost-Push NKPC-W	AR	0.031	0.991	Beta(0.5, 0.2)
	sd	0.274	0.555	Invgamma(0.1, 2)
	MA	0.640	0.991	Beta(0.5, 0.2)

Shock Process Estimates on SW Data.

Cont'd

Metric	RF	Sticky
Optimised Log-Likelihood	4372.986	2335.606
Combined Shock Variance	63.883	112.735

Shock Process Estimates on SW Data: Key Statistics.

Summary and Next

Summing up:

- ▶ Propose a flexible, structured framework for departing from FIRE building on the reduced-form model in [Kohlhas and Warther \(2021\)](#), consistent with macro, micro-foundations, and empirical evidence.
- ▶ Implementation of the RF expectations in the Sequence Space and RF Solvent.
- ▶ Application to HANK models in the sequence space, covering implications for **macro-transmission and dynamics**:
 1. Develop and solve under alternative expectation processes (FIRE, Sticky, RF) a One-Asset and a Two-Asset HANK, replicating closely features of [Auclert et al. 2021](#).
 2. Micro-jumps and Macro-humps under alternative expectation processes, and how RF mitigates the dampening effect of sticky expectations on amplification.
 3. *Partial information* estimation of the RF expectations parameters by IRF-matching to the empirical IRFs.
 4. *Likelihood-Based* estimation of the aggregate shock processes as in [Smets and Wouters \(2007\)](#) across FIRE, Sticky, and RF expectations.

Cont'd

Next:

- ▶ Refine partial and full info estimation; speed up RF solvent.
- ▶ Forecast error simulations and decompositions, similar to BC ones.
- ▶ More work on the Bauer & Swanson IRF matching
- ▶ Fully Bayesian estimation
- ▶ ... policy questions?

Thank you!

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Micro-Jumps and Macro-Humps

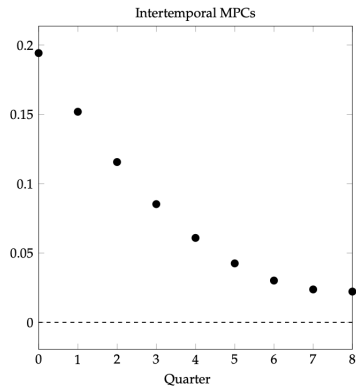
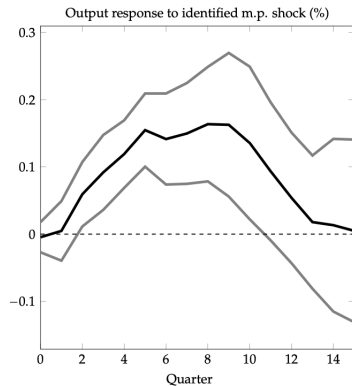


Fig. 1 in MJMH by [Auclert et al., 2020](#)

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Assumptions for Recursive Forecast LoM

Ensure internally rational agents (Adam and Marcet, 2011)

1. No Bias along the Steady State path

$$c := \frac{1}{1+\delta} \int a_j P(dj) = \frac{1}{1+\delta} \gamma^{x_{ss}} \rightarrow \int a_j P(dj) = \gamma^{x_{ss}}$$

2. A Law of Iterated Expectations holds, $\forall t, s$ with $s = t - r, r \geq 0$, so that agents do not expect or forecast their own forecast errors (these are orthogonal to their information set at the time expectations are formed):

$$\bar{f}_s[\bar{f}_t x_{t+k}] = \bar{f}_s[x_{t+k} - \epsilon_{t+k|t}] = \bar{f}_s x_{t+k} = \bar{f}_s x_{s+r+k}$$

3. For large enough T , agents expect the forecast variable to return to the steady state Bardoczy and Guerreiro, 2024:

$$\exists T \in \mathbb{R}_+ : \forall t \leq T \quad \bar{f}_t x_T = x_T = x_{ss} = \bar{f}_{ss} \bar{x}_{ss}$$

4. News revision coefficient $\delta > 0$, which is sufficient to guarantee *stability* of the law of motion for the forecast in the sequence space around the steady state forecast.

The Linear \mathcal{K}_t Mapping

$$\mathcal{K}_0 = \begin{pmatrix} 1 & 0 & 0 & 0 & \cdots & 0 \\ \frac{-\gamma}{1+\delta} & \frac{1}{1+\delta} & 0 & 0 & \cdots & 0 \\ \frac{-\gamma}{1+\delta} & 0 & \frac{1}{1+\delta} & 0 & \cdots & 0 \\ \frac{-\gamma}{1+\delta} & 0 & 0 & \frac{1}{1+\delta} & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \frac{-\gamma}{1+\delta} & 0 & 0 & 0 & \cdots & \frac{1}{1+\delta} \end{pmatrix}$$

More generally, we show for $\tau \leq T$

$$\Lambda_k = \begin{pmatrix} I_{k+1} & 0_{(k+1) \times (T-(k+1))} \\ A_k & B_k \end{pmatrix}$$

$$\text{where } A_{k(i,j)} = \begin{cases} \frac{-\gamma \delta^{k-j}}{(1+\delta)^{k-j+1}} & \text{for } j = 0, 1, \dots, k \\ 0 & \text{otherwise} \end{cases},$$

$$B_{k(i,j)} = \begin{cases} \sum_{l=0}^k \frac{\delta^l}{(1+\delta)^{l+1}} & i = j \\ 0, & i \neq j \end{cases}$$

Faster Implementation

- Practically, faster to work with just the FIRE Fake-News matrix, a more primitive object than the Jacobian.

$$\hat{J}_{t,r} = \frac{\partial Y_{i,t}}{\partial X_s} = \frac{dY_{i,t}}{dX_s} = \mathcal{F}_{t-r,0} + \sum_{\tau=0}^{t-1} \sum_{s=0}^T \mathcal{F}_{t-\tau,s-\tau} \mathcal{K}_{\tau,s,r} + \sum_{s=t+1}^T \mathcal{F}_{0,s-t} \mathcal{K}_{t,s,r}$$

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Mutual Fund and Asset Pricing

- Funds clearing:

$$(1 + \delta q_t)B_{t-1} + (p_t^k + d_t^k) + (p_t^y + d_t^y) = (1 + r_t^l)L_{t-1} + (1 + r_t^a + \xi)A_{t-1}$$

- Implying the asset pricing/no arbitrage conditions

$$E_t r_{t+1} = E_t r_{t+1}^a = E_t [r_{t+1}^l + \xi]$$

$$E_t \left(\frac{d_{t+1}^i + p_{t+1}^i}{p_t^i} - (1 + r_{t+1}) \right) = 0$$

$$E_t \left(\frac{1 + \delta q_{t+1}}{q_t} - (1 + r_{t+1}) \right) = 0$$

Steady State Tables

Financial Side: Aggregate Wealth (% GDP)

Agg. Wealth	(p.a)	3.8199999999999985
Agg. Illiquid Wealth	(p.a)	3.58641
Agg. Liquid Wealth	(p.a)	0.23358999999999858
Equity Value K Firm	(p.a)	2.23
Equity Value Y Firm	(p.a)	1.1699999999999988
Gov. Bonds Stock	(p.a)	0.33705
Maturity Structure of Gov Debt	(p.a)	5.000000000000001
Markup		1.0621348911311732

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Steady State Tables

Wealth and Income Distribution (% GDP)

Illiquid Wealth:

Group 1 Illiquid Asset Holdings	2.7 % Agg. Ill. Wealth
Group 2 Illiquid Asset Holdings	7.01 % Agg. Ill. Wealth
Group 3 Illiquid Asset Holdings	7.01 % Agg. Ill. Wealth
Group 4 Illiquid Asset Holdings	13.01 % Agg. Ill. Wealth
Group 5 Illiquid Asset Holdings	12.21 % Agg. Ill. Wealth
Group 6 Illiquid Asset Holdings	58.06 % Agg. Ill. Wealth

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Steady State Tables

Other Steady State Variables

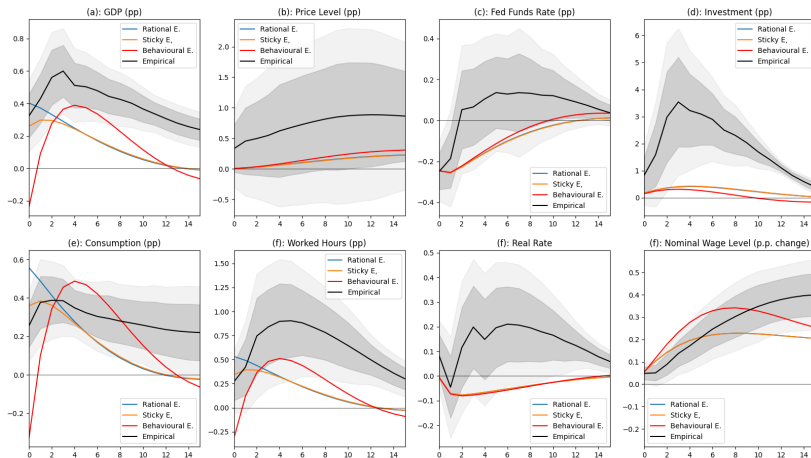
Final Good Firm Markup	1.0621348911311732
r star (p.a)	5.0 %
Return on Illiquid Asset (p.a)	5.0 %
Return on Liquid Asset (p.a)	-1.5 %

Time Preferences of Wealth Distribution Groups (Endogenous)

Discount Factor Group 2 (beta_1)	0.905101093412752
Discount Factor Group 2 (beta_2)	0.9191010934127518
Discount Factor Group 3 (beta_3)	0.933101093412752
Discount Factor Group 4 (beta_4)	0.9461010934127517
Discount Factor Group 5 (beta_5)	0.950101093412752
Discount Factor Group 6 (beta_6)	0.9751010934127519
Average Time Preferences	0.9794749985372796

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Bauer & Swanson IRFs (Unmatched)



One-Asset Model vs Bauer and Swanson (2023) Proxy-VAR IRFs,
RF-expectations and nested solvents

How to build the LLF $\mathcal{L}(\mathbf{Y}|\Theta)$ in Sequence Space

State-Space \rightarrow Kalman Filter.

How about the Sequence-Space? [Auclert et al. 2021](#)

Sequence Space Covariance Matrix ($T \times T \times N_{obs}$)

1. Simulation based from 1 std pulse at origin

$$d\mathbf{U} = \underbrace{\left[\left(\mathbf{J}\mathcal{K}_0 + \sum_{h=1}^T R_h(\mathcal{K}_h - \mathcal{K}_{h-1}) \right) \mathbf{S}_X^{-1} \mathbf{S}_U \right]^{-1} \mathbf{F}_Z d\mathbf{Z}_k}_{\text{Gen. Eq. Jacobian with Flexible Deviations from FIRE}}$$

2. Using $\text{MA}(\infty)$ representation of the Gaussian shock process

$$d\mathbf{Z}_t = \sum_j^{\infty} M_Z^j d\tilde{\mathbf{Z}}_{t-j}$$

3. Get covariance matrix Σ of \mathbf{Y} from above
4. Sequences \mathbf{Y} are multivariate normal with mean zero and covariance matrix Σ .