Paper 7 Regional Economics Supervision 1: Dixit-Stiglitz Model and Applications in *New Trade Theory* (Home Market Effect)

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Outline

Rationale for MP

- Limitations of fierce (technically perfect) competition: price-setting powers and price variation; profits and markups.
- Obtain PC as a limit or special case (more on this later)
- Product differentiation and preference for variety: spatial, sectoral, etc phenomena.
- New Trade or DSK Models in Regional, Geographic, and Development Ecoomics. (Home Market Effect!)
- From macroeconomic theory perspective, nominal rigidities/Phillips Curve in NK models and IS-LM require price-setting power.

Differentiation and Market Power

- In PC case, a firm "setting" price *above* marginal cost gets wiped out. Why?
  1. The (ex-post representative) firm is small and cannot influence (equilibrium) aggregate price through output choices: it faces a flat inverse demand curve.
  2. Goods are perfect substitutes or homogenous. Consumer swaps for cheaper producer.
- Goal is to get a downward sloping demand curve *at the firm level*. Different ways of achieving this.
- Here, popular approach: **preference for variety**
  - Goods/outputs are *imperfect* substitutes AND firms know this.
  - Relaxing (1)-(2).
  - With preferences for variety and downward sloping demand curve, a (new) firms can always set up production of a new good and make profits. It will do so as long as the profits cover fixed labour costs. (What happens without IRS? How about with perfect substitutes?).
- Dixit-Stiglitz Preferences (CES utility functions).

Cont'd: Main Aspects/Applications[1]

- Variety encompasses any specialised aspects of interest → spatial location...
- HH: downward sloping demand for a good of variety (read: produced by firm) i.
  1. Substitution away from the "i-th variety" good due to markup over PC price is offset by preference for variety.
  2. Quantification/estimation.
  3. Some limitations of the demand function omitted by textbook treatments.
- Rational Firms optimal pricing: constant markup over marginal costs: $p_i = \mu C'(q_i), q_i = c_i(p_i)$. *Graph*.
- Pulling together: preference for variety and downward-sloping DCs → there is equilibrium output differention + output/production of any variety (location!) depends on the demand curve... key to Home Market effect. Q: How about no love for variety/PC?

[1] Extras: (1) Natural Unemployment/Output Loss. (2) Aggregate demand (pecuniary) spillovers. Used in models of development economics (e.g. Acemoglu, 2003).

CES utility

- Let $\mathbf{c} = \{c_i\}_{i \in N}$ a consumption bundle. Preferences are represented by a utility function U: $\mathbb{R}^n_+ \to \mathbb{R}$:

$$U(\mathbf{c}) = \left(\int_{i \in N} c(i)^{\frac{\sigma-1}{\sigma}} di \right)^{\frac{\sigma}{\sigma-1}} \approx \left(\sum_{i \in N} c_i^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}$$

- Where $\rho = \frac{\sigma-1}{\sigma}$ is an (inverse) substitution parameter. More on this later.
- Parametrisation determines (at an equilibrium) *Complements vs Substitutes* nature of goods (and anything in between, and limit cases).
    1. $\rho \to -\infty \Rightarrow \sigma(\rho) \to 0$: (Leontief) complements.
    2. $\rho \to 0 \Rightarrow \sigma(\rho) \to 1$: Cobb-Douglass (imperfect substitutes).
    3. $\rho \to 1 \Rightarrow \sigma(\rho) \to \infty$: Perfect/Linear substitutes (linear utility...).
    4. Generally, we let $\sigma(\rho) > 1$ (gross or imperfect substitutes... req: $\rho \in (0, 1)$)

CES utility

- Two questions from problem set:
  1. Suppose preference for variety with goods as gross substitutes ($\sigma > 1$). Are *more balanced* bundles preferred? How do we show this?
  2. What's the role of the parameter ρ (or σ) in driving this?
- Solutions:
  1. Multiple approaches accepted for this supervision.
     1.1 Discretisation and objective evaluation on feasible set.
     1.2 Noting monotonicity, concavity, hence solving consumer problem for *unique* solution – which is a bundle with equal consumptions.
     1.3 In any case, it's *concavity* of preferences driving the result.
  2. This should inform your answer to 2. Higher values of ρ (as it converges $\rho \to 1$) imply lower utility from a balanced or mixed consumption *relative* to specialised one (CES utility becomes linear and MU constant).

Cont'd

- Key for either approach, conceptually, is relationship between ρ and concavity ($=$ bowed out ICs $=$ how much preferred mixing/combinations are, recall from Paper 1 last year).
- Rescale utility by $u(\mathbf{c}) = U(\mathbf{c})^{\frac{\sigma-1}{\sigma}}$.
- $u'(c_i) = \frac{\sigma-1}{\sigma} c_i^{-1/\sigma}$ positive MU, i.e. monotonic prefs.
- $u''(c_i) = -\frac{\sigma-1}{\sigma^2} c_i^{\frac{-1-\sigma}{\sigma}}$
- With *gross substitutes assumption* ($\sigma > 1$ or $\rho \in (0,1)$), concave function $u'' < 0 \rightarrow$ diminishing marginal utility!
- Note: $-u''(c)c/u'(c) = 1/\sigma$
- Recall from P1A/Micro: with diminishing marginal utlity, consuming an extra unit of the *same* variety yields lower increase in utility than allocating it to another variety.
- This is why linear combinations/mixing is preferred with $\sigma > 1$.
- What if $\rho \rightarrow 1$? MU constant (u"=0). Plug in in U to verify for $\rho \in (0,1)$ vs $\rho = 1$ (linear case).

Cont'd

- As said, two approaches both exploiting concavity (key to role of ρ substitution parameter).
- Here I show you my approach.
- Establishes that with equal prices and $\rho \in (0, 1)$, consuming identical quantities (i.e. perfectly balanced consumption) is optimal $\rightarrow$ by construction of the solution, utility at such point attains a maximum over the feasible set (=better than any other combination).
- We get the demand function for CES for free, which we need for monopolistic competition.

My Solution

- Because for $\rho \in (0,1)$ the CES utility function is monotonic and strictly concave (hence strictly quasi-concave), there is a unique solution to the consumer problem.
- By virtue of the same properties and Lagrange Theorem, necessary and sufficient for **c** to be an optimum/equilbrium for a given price vector is that it satisfies the FOCs for the Lagrangian of the problem.
- At an interior solution, for any two consumption varieties/locations (c_i, c_j)

$$\left(\frac{c_i}{c_j}\right)^{-\frac{1}{\sigma}} = \frac{p_i}{p_j}$$

- Assumption $p_i = p_j = 1 \quad \forall i, j \rightarrow c_i = c_j \quad \forall c_i, c_j$.
- Optimum is a perfectly balanced consumption. Plug in budget constraint to get $c_i = Y/N \quad \forall i$. QED.
- Concavity is key. How about linear case with $\sigma \rightarrow \infty$?

Some problems of CES demand (Extra)

- Optimal ratios/margins between two goods are independent of third alternatives (IIA) $\rightarrow$ credible?
- Proportional substitution: suppose price of a third alternative c_z increases, so that demand relative to both c_i and c_j drops. Their ratio is still constant, hence substitution is proportional.

Monopolistically Competitive Firms

- We use the above solution (with gross substitutes assumption) to get the downward sloping demand faced by the firm.
- A rational firm knows this and sets price optimally (marginal revenue = marginal costs) at a markup over the marginal costs.
- The markup is closely related to the substitution parameter.

CES Downward Sloping Demand Curve

At an optimum $\mathbf{c}(\mathbf{p})$, the optimisation condition holds $\left(\frac{c_i}{c_j}\right)^{-\frac{1}{\sigma}} = \frac{p_i}{p_j}$ Rearrange to get

$$p_i^{\sigma} c_i p_j^{-\sigma} = c_j$$

$$p_i^{\sigma} c_i p_j^{1-\sigma} = p_j c_j$$

Summing over $j \neq i$, and letting M nominal consumption (expenditure on consumption):

$$p_i^{\sigma} c_i p_j^{1-\sigma} = M$$

Defining the *Price Index/Level or Price Aggregator*

$$P = \left(\sum_{j \neq i} p_j^{1-\sigma}\right)^{\frac{1}{1-\sigma}}$$

CES Downward Sloping Demand Curve

We can rewrite the optimality condition as

$$p_i^{\sigma} c_i P^{1-\sigma} = M$$

And hence we obtain the CES demand function (known to firms...):

$$c_i = \frac{M}{P}\left(\frac{p_i}{P}\right)^{-\sigma} = m\left(\frac{p_i}{P}\right)^{-\sigma}$$

1. Price elasticity of consumption: $\epsilon = \frac{\partial \ln c_i}{\partial \ln p_i} = -\sigma$
2. Aggregate demand spillovers m: used in multiple equlibrium models of development (big-push, see Acemoglu 2003):
3. Since m is real expenditure, and this is equal to real income (in a static model), with inelastic labour supply and common technology Y=L:

$$c_i = \left(\frac{p_i}{P}\right)^{-\sigma} y = \left(\frac{p_i}{P}\right)^{-\sigma} wl = \left(\frac{p_i}{P}\right)^{-\sigma} wY$$

4. Demand for a good increases when aggregate output is higher (via income/wages).

Dixit-Stiglitz Monopolistic Competition

- Preferences for variety $\rightarrow$ downward sloping (not flat) demand curves and differentiated output.
- Firms anticipate this. How do they set prices?
- With preferences for variety and downward sloping demand curve, a (new) firms can always set up production of a new good and make profits (by charging a markup). Why?
- It will do so as long as the profits cover fixed labour costs. (What happens without IRS? How about with perfect substitutes?).

Dixit-Stiglitz Monopolistic Competition

- Prices set for profit maximisation. Show graphically with downward sloping demand curve.
- Mathematically: we will assume later that there are fixed costs wf determining entry/exit decisions. Once they are paid (i.e. conditionally on profit at the optimal price, net of fixed costs, being non-negative) decisions are based on variable costs only. With a linear technology $q_i = \phi l$, and requiring in gen.eq. $q_i = c(p_i)$:

$$\max_{p_i} p_i \left(\frac{M}{P}\right) \left(\frac{p_i}{P}\right)^{-\sigma} - \frac{w}{\phi} \left(\frac{M}{P}\right) \left(\frac{p_i}{P}\right)^{-\sigma} - wf$$

Cont'd

$$\max_{p_i} p_i \left(\frac{M}{P}\right)\left(\frac{p_i}{P}\right)^{-\sigma} - \frac{w}{\phi}\left(\frac{M}{P}\right)\left(\frac{p_i}{P}\right)^{-\sigma} - wf$$

$$\max_{p_i} M \left(\frac{p_i}{P}\right)^{1-\sigma} - \frac{w}{\phi}\left(\frac{M}{P}\right)\left(\frac{p_i}{P}\right)^{-\sigma} - wf$$

FOC:

$$(1-\sigma)\frac{M}{P}\left(\frac{p_i}{P}\right)^{-\sigma} + \sigma\frac{w}{\phi}\frac{M}{P^2}\left(\frac{p_i}{P}\right)^{-\sigma-1} = 0$$

$$(1-\sigma)\left(\frac{p_i}{P}\right)^{-\sigma} + \sigma\frac{w}{\phi}\frac{1}{P}\left(\frac{p_i}{}\right)^{-\sigma-1} = 0$$

$$(\sigma-1)p_i = \sigma\frac{w}{\phi} \rightarrow p_i = \frac{\sigma}{\sigma-1}\frac{w}{\phi}$$

Cont'd

- The equilibrium price satisfies:

$$p_i = \frac{\sigma}{\sigma - 1} \frac{w}{\phi}$$

- It is the *markup condition*. The term w/phi represents the marginal cost of increasing (equilibrium) output by one unit. $=$ marginal cost of a unit of labour (w) $\times$ marginal cost of a unit of output" in terms of extra labour $(1/\phi)$.
- Now that we know what σ is and how it relates to pref for variety: what happens with *gross substitutes*?
- What happens with *perfect substitutes*?
- Could estimate markups by regression (OLS or Fixed Effects) if we can observe "true" marginal costs.

Toward Applications: General Equilibrium in a Closed Economy (Dixit-Stiglitz-Krugman Model)

- To fix some ideas, we begin by solving the model in a closed economy (no trade).
- Households have CES preferences. Firms set prices optimally and have access to an IRS technology with labour costs $l_i = f + \frac{q_i}{\phi}$. At an equilibrium:

$$p_i = \frac{\sigma}{\sigma - 1}\frac{w}{\phi} = \bar{p}$$

- Goods market clear

$$q_i = L_i c_i$$

- Zero Profits/Free Entry:

$$p_i q_i - w\frac{q_i}{\phi} - wf = 0 \rightarrow q_i = \frac{f}{\frac{p_i}{w} - \frac{1}{\phi}}$$

- Labour markets clear

$$L = n(f + \frac{q_i}{\phi})$$

Cont'd

- With preferences for variety and downward sloping demand curve, a (new) firms can always set up production of a new good and make profits. It will do so as long as the profits cover fixed labour costs. (What happens without IRS? How about with perfect substitutes?).
- Hence the zero profit condition will pin down the equilibrium number of firms/varieties.

$$p_i q_i - w(\frac{q_i}{\phi} + f) = 0$$

$$p_i q_i - \frac{wL}{n} = 0$$

$$\frac{\sigma}{\sigma - 1} \frac{1}{\phi} q_i = \frac{L}{n}$$

- At zero profits (when the last firm enters), output per firm is unrelated to the number of firms. We see that as L increases, then so must n in equilibrium: innovation/diversification/more firms.

Toward Applications: General Equilibrium in a Closed Economy (Dixit-Stiglitz-Krugman Model)

- Paul Muad'Dib attacks spice harvesters (Iceberg Costs) – $\tau > 1$ units to be produced for a one net unit sold.
- Now marginal costs are higher. At the zero profit condition:

$$\frac{\sigma}{\sigma - 1}\frac{\tau}{\phi}q_i = \frac{L}{n}$$

Krugman 1980 Trade Model: Home Market Effect

- Now two regions producing a differentiated good, $N = n^h + n^f$ (endogenous)
- There are again IRS (due to fixed costs to setting up production in a location). Hence firms (of any size) will choose to locate in one region only. Also, lower average costs from producing in a larger market...
- There are iceberg costs:
  1. A constant fraction of goods gets "destroyed" – to deliver q_i units produce and ship τq_i. Labour input at equilibrium q_i then $l_i = f + \tau \frac{q_i}{\phi}$
  2. Because this is passed to prices charged abroad, it is the same as a flat/linear tax.
- (Marginal) cost and hence profit from producing the good in a location thus depends, via trade costs, on faced domestic relative to foreign demand.
- With constant markups over marginal costs, firm will choose to locate in markets with a larger domestic demand.

Cont'd

- Why? Suppose wages are the same in both regions (perfectly elastic labour supply).
- Then with trade costs, the costs of producing in any region are larger the larger is foreign demand relative to domestic one in that region.
- But then, for a firm to locate in the smaller domestic market, it must face less competition. Hence the number of firms setting up and exporting from the smaller market is smaller. That is, we have $n^h > n^f$.

Model

- Let L_h, L_f, n_h, n_f respectively the sizes of labour forces/mass of households and number firms home and abroad. Labour input is as usual.
- A representative home and foreign consumers/households consumes differentiated good from both regions:

$$U(c^{h,f}) = \left(\sum_{i \in N} c_i^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}$$

- Firms have profit functions from selling domestically and abroad (at different prices):

$$\pi = p_i^h L_h c_i^h + p_i^f L_f c_i^f - w \frac{L_h c_i^h}{\phi} - \tau w \frac{L_f c_i^f}{\phi} - wf$$

- Note we are imposing the same wage, as a simplification.

Equilibrium I: Goods Market

- Quantity Demanded/Consumption of each variety for home region household:

$$c_i = \frac{M}{PH}\left(\frac{p_i}{PH}\right)^{-\sigma} = \frac{w}{PH}\left(\frac{p_i}{PH}\right)^{-\sigma}$$

- Firm's prices for domestic and foreign household (in any region)

$$p_i^h = \frac{\sigma}{\sigma - 1}\frac{w}{\phi} \qquad p_i^f = \frac{\sigma}{\sigma - 1}\frac{\tau w}{\phi}$$

- With same marginal cost per unit of output, iceberg costs ($\tau > 1$) imply optimal price slightly larger. Other than that, all domestically produced goods and foreign produced goods are priced symmetrically in "home" region.
- Combining the above, depending on whether demanded good is produced domestically or abroad, at home equilibrium domestic and imports aggregate consumptions are

$$q^{hh} = q_i^h = \frac{L_h w}{PH}\left(\frac{p_i^h}{PH}\right)^{-\sigma} \qquad q^{hf} = q_i^f = \frac{L_h w}{PH}\left(\frac{p_i^f}{PH}\right)^{-\sigma}$$

Equilibrium II: Labour Market and Free Entry Conditions

- At home, labour market equilibrium requires (under assumption of IRS so that firms are either home or domestic producers):

$$L_h = n^h \left(f + \frac{q^{hh}}{\phi} + \tau \frac{q^{fh}}{\phi} \right)$$

- Where q^{fh} i.e. exports is counterpart to q^{hf} for the foreign region $q^{fh} = L_f c_i^f$.
- Free entry:

$$\pi = p_i^h q^{hh} + p_i^f q^{fh} - w \frac{q^{hh}}{\phi} - \tau w \frac{q^{fh}}{\phi} - wf = 0$$

- Trade balance condition

$$n^h p^h q^{hh} + n^f p^f q^{hf} = wL_h$$

$$n^f p^h q^{ff} + n^h p^f q^{fh} = wL_f$$

- Mirroring conditions for the other region.

Solution

With some algebra, one can show that equilibrium conditions imply:

$$\frac{n_h}{n_f} = \frac{\frac{L_h}{L_f} - \tau^{1-\sigma}}{1 - \frac{L_h}{L_f}\frac{1}{\tau^{\sigma-1}}}$$

.

$$\frac{n_h}{n_f} = \frac{\tau^{\sigma-1}\frac{L_h}{L_f} - 1}{\tau^{\sigma-1} - \frac{L_h}{L_f}}$$

.

For $\tau^{1-\sigma} \leq L_h/L_f \leq \tau^{\sigma-1}$, then $L_h/L_f \geq 1$ and trade costs $\tau > 1$ deliver the Home Market Effect.

The "Equilibrium Number of Firms" Idea

1. Firms are free to enter, set up, and leave any region. The zero profit condition tells us that firms will enter the region until it is profitable to do so (i.e. until the free entry condition is met).
2. The presence of Iceberg transport costs reduces the profits made my producing at home and selling abroad. Similarly, they hamper the economies of scale achieved by producing more for foreign consumption. Economies of scale (fixed costs), further, imply that production location will be indivisible.
3. Hence, profits will be larger for firms located in regions with relatively larger domestic markets. But then the zero profit condition/free entry condition requires more competition (which lowers profits) in such region, i.e. more producers/varieties of goods.
4. Monopolistic competition of course is key: with downward sloping demand curves, there are profits to make by setting up production of a new variety of goods.